

Multigluon correlations in JIMWLK

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Abstract

We discuss applications of the JIMWLK renormalization group equation to multigluon correlations in high energy collisions. This includes recent progress in computing the energy dependence of higher point Wilson line correlators from the JIMWLK renormalization group equation. We find that the large N_c approximation used so far in the phenomenological literature is not very accurate. On the other hand a Gaussian finite N_c approximation is surprisingly close to the full result. We also discuss correlations at large rapidity separations, relevant for the “ridge” correlations observed in experiments.

Keywords: JIMWLK, CGC, dihadron correlations

1. Introduction

The physics of high energy hadronic or nuclear collisions is dominated by the gluonic degrees of freedom of the colliding particles. These small x gluons form a dense nonlinear system that is, at high enough \sqrt{s} , best described as a classical color field and quantum fluctuations around it. The color glass condensate (CGC, for reviews see [1]) is an effective theory developed around this idea. It gives an universal description of the small x degrees of freedom that can equally well be applied to small x DIS as to dilute-dense (pA or forward AA) and dense-dense (AA or very high energy pp) hadronic collisions. The nonlinear interactions of the small x gluons generate dynamically a new transverse momentum scale, the saturation scale Q_s , that grows with energy. At high enough energy the color glass condensate is thus a one-scale system, characterized by a dominant momentum scale Q_s that is hard enough to justify a weak coupling calculation. The scale Q_s dominates both the gluon spectrum and multigluon correlations. The nature of a unique saturation scale as both the typical gluon transverse momentum and as the correlation length $1/Q_s$ differentiates the CGC qualitatively from the high- x part of the wavefunction.

The most convenient parametrization of the dominant gauge field is in terms of Wilson lines that describe the eikonal propagation of a projectile through it. The Wilson lines are drawn from a probability distribution, whose dependence on rapidity is described by the JIMWLK renormalization group equation. It reduces, in a large N_c and mean field approximation, to the BK [2] equation and further, in the dilute linear regime, to the BFKL one.

2. Correlations in a dilute-dense collision

One of the more striking signals of saturation physics at RHIC is seen in the relative azimuthal angle $\Delta\varphi$ dependence of the dihadron correlation function, where the $\Delta\varphi \approx \pi$ back-to-back peak is seen to be suppressed in dAu-collisions compared to pp collisions at the same kinematics [4]. The CGC description of this correlation starts from a large x parton pair propagating eikinally through the target. For a dilute target momentum conservation causes

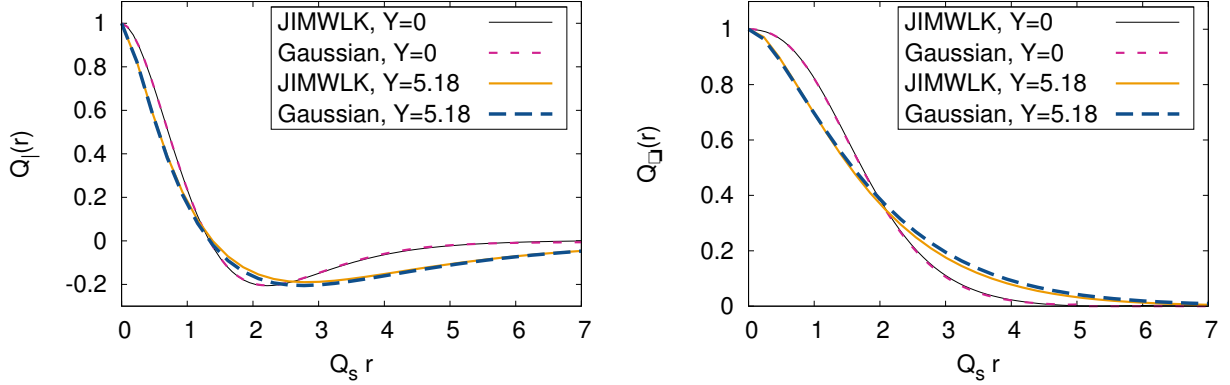


Figure 1: The JIMWLK result for the quadrupole correlator compared to the Gaussian approximation. Shown are the initial condition (MV model) at $y = 0$ and the result after 5.18 units of evolution in rapidity, for the “line” (left) and “square” (right) coordinate configurations. Figures from Ref. [3].

a peak in the correlation function at $\Delta\varphi \approx \pi$, whereas for a dense one the intrinsic transverse momentum, of the order of Q_s , causes the peak to disappear for semihard momenta. To calculate the matrix element for this process one needs target expectation values of products of Wilson line operators, such as the dipole and the quadrupole

$$\hat{D}(\mathbf{x}_T, \mathbf{y}_T) = \frac{1}{N_c} \text{Tr} U(\mathbf{x}_T) U^\dagger(\mathbf{y}_T) \quad \hat{Q}(\mathbf{x}_T, \mathbf{y}_T, \mathbf{u}_T, \mathbf{v}_T) = \frac{1}{N_c} \text{Tr} U(\mathbf{x}_T) U^\dagger(\mathbf{y}_T) U(\mathbf{u}_T) U^\dagger(\mathbf{v}_T). \quad (1)$$

For practical phenomenological work it would be extremely convenient to be able to express these higher point correlators in terms of the dipole, which is straightforward to obtain from the BK equation. In the phenomenological literature so far [5] this has been done using a “naive large N_c ” approximation where the quadrupole is assumed to be simply a product of two dipoles. A more elaborate scheme would be a “Gaussian” approximation (“Gaussian truncation” in [6]), where one assumes the relation between the higher point functions and the dipole to be the same as in the (Gaussian) MV model. The expectation value of the quadrupole operator in the MV model has been derived e.g. in Ref. [7].

In Ref. [3] the validity of these approximations was studied by comparing them numerically to the solution of the JIMWLK equation. As studying the full 8-dimensional phase space for the quadrupole operator would be cumbersome, we have concentrated on two special coordinate configurations. The “line” configuration is defined by taking $\mathbf{u}_T = \mathbf{x}_T$ and $\mathbf{v}_T = \mathbf{y}_T$, with $r = |\mathbf{x}_T - \mathbf{y}_T|$ and the “square” by taking $\mathbf{x}_T, \mathbf{y}_T, \mathbf{u}_T, \mathbf{v}_T$ as the corners of a square with side r .

Our most important results in Ref. [3] for the quadrupole expectation value are shown in Figs. 1 and 2, with a comparison of the initial and evolved (for 5.18 units in y) results to the approximations. The MV-model initial condition $y = 0$ satisfies the Gaussian approximation by construction. Figure 1 shows that the Gaussian approximation is still surprisingly well conserved by the evolution. A possible explanation for this based on the structure of the JIMWLK equation has recently been proposed in [8]. The naive large N_c approximation, on the other hand, fails already at the initial condition, as shown in Fig. 2. This stresses the importance of the various SU(3) group structure constraints violated in this approach. Crucially for the phenomenological consequences, even the characteristic length/momentum scale differs by factor ~ 2 from the actual result.

This result does not yet fully address the effect on the measurable dihadron cross section. For that one must convolute the Wilson line operators with the $q \rightarrow qg$ splitting wavefunction. This nontrivial numerical task is still work in progress, discussed in this conference in [9].

3. Unequal rapidity correlations and the ridge

Calculating multigluon correlations in a collision of two dense gluonic systems requires including the JIMWLK evolution for both of the colliding projectiles (for a more formal discussion see [10]). Of particular phenomenological

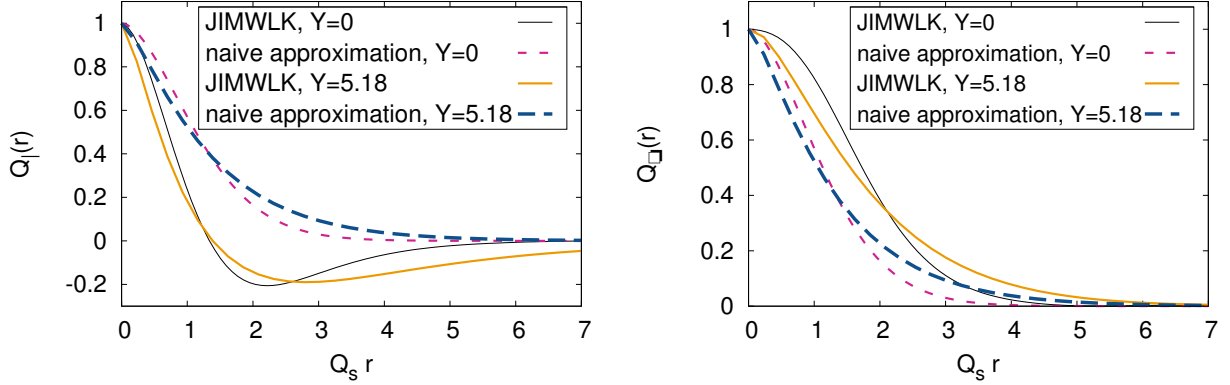


Figure 2: The JIMWLK result for the quadrupole correlator compared to the “naive large N_c ” approximation. Shown are the initial condition (MV model) at $y = 0$ and the result after 5.18 units of evolution in rapidity, for the “line” (left) and “square” (right) coordinate configurations. Figures from Ref. [3].

interest are correlations at large rapidity separations, because they are directly responsible for the “ridge” long range rapidity correlations observed in nucleus-nucleus and high multiplicity proton-proton collisions. In spite of this, most practical work on the JIMWLK equation has so far concentrated on correlations of Wilson lines at one rapidity.

The existing applications of the CGC framework to calculations of the ridge [11, 12] have used a simplified argument based on the MV model, where the change in the effective color charge in one infinitesimal step in rapidity is independent of the color charge. This leads to an unequal rapidity correlation function of Wilson line operators which is independent of the rapidity separation between the produced particles. Consider for example the following correlation function, which is similar to the one appearing in the calculation of the ridge correlation in the k_T -factorized approximation [11]:

$$(N_c^2 - 1) \left[\frac{\langle \hat{D}(\mathbf{k}_T)_y \hat{D}(\mathbf{k}_T)_{y+\Delta y} \rangle}{\langle \hat{D}(\mathbf{k}_T)_y \rangle \langle \hat{D}(\mathbf{k}_T)_{y+\Delta y} \rangle} - 1 \right], \quad (2)$$

where $\hat{D}(\mathbf{k}_T)$ is the Fourier-transform of the dipole operator(1). In the MV model calculation of [11] this quantity would be one independently of Δy . This naturally leads to a very long range correlation in rapidity between produced gluons, as seen in [11, 12]. Genuine JIMWLK evolution should be expected to decorrelate the Wilson line operators, at a characteristic rapidity scale $\Delta y \sim 1/\alpha_s$. In Fig. 3 we show preliminary results for the correlation function (2). On the left is plotted the correlation function itself at different rapidity separations Δy . There is only a mild dependence on the transverse momentum k_T , as expected from the MV model calculation. The decorrelation seems, however, to be relatively fast. This is further quantified on the right, where we show the result for the decorrelation speed ζ in an exponential decay fit

$$\frac{\int d^2 \mathbf{k}_T k_T^4 \left[\langle \hat{D}(\mathbf{k}_T)_y \hat{D}(\mathbf{k}_T)_{y+\Delta y} \rangle - \langle \hat{D}(\mathbf{k}_T)_y \rangle \langle \hat{D}(\mathbf{k}_T)_{y+\Delta y} \rangle \right]}{\int d^2 \mathbf{k}_T k_T^4 \hat{D}(\mathbf{k}_T)_y} \sim \exp\{-\zeta y\}. \quad (3)$$

Here the weighting with k_T^2 is chosen to match the unintegrated gluon distribution $\sim k_T^2 D(k_T)$ appearing in a k_T -factorized calculation of gluon production. In Fig. 3 the decorrelation speed ζ , explicitly $\sim \alpha_s$, is compared to the natural evolution speed λ , defined by $\lambda \equiv d \ln Q_s^2 / dy$, measured in the same JIMWLK simulation. The decorrelation is seen to be surprisingly fast. What is more worrying, there seems to be a slight logarithmic dependence on the infrared cutoff given by the system size L , which persists over a variety of different initial conditions, lattice sizes, values at which the running coupling is frozen in the infrared and other parameters varied in the calculations. At the moment there appears to be no natural interpretation of this dependence, leaving the implications for ridge phenomenology uncertain.

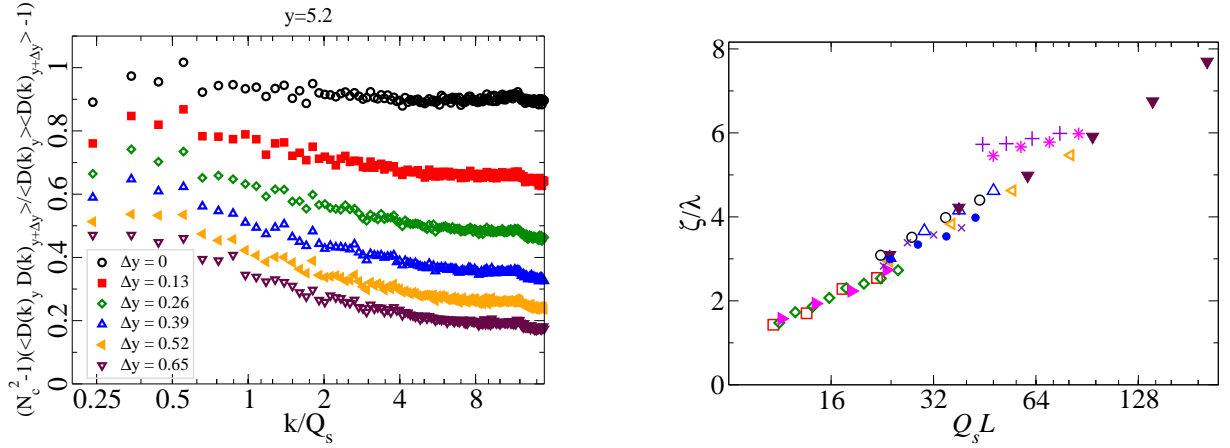


Figure 3: Left: Unequal rapidity correlation (2) as a function of k_T/Q_s for different rapidity separations. Right: Dependence of the decorrelation speed ζ on $Q_s L$, where L is the linear size of the system.

4. Conclusion

We have here argued that multiparticle correlations provide unprecedented experimental insight into the details of nonlinear QCD dynamics at small x . In particular, they make it necessary to go beyond the mean-field BK equation and use the full JIMWLK equation. Azimuthal angle correlations of particles produced in a collision with a dilute probe and dense, saturated target are sensitive to multipoint functions of Wilson lines in the target wavefunction. We have shown that a “naive large N_c ” approximation used in the literature, where the quadrupole operator is assumed to be a simple product of dipoles, is far from the true finite N_c result. A Gaussian approximation, based on the MV model, turns out to be surprisingly close to the result from JIMWLK evolution. We have also discussed preliminary results on unequal rapidity correlations which are needed for a proper CGC calculation of the “ridge” correlation in high energy collisions. We have shown that at least some observables display a problematic infrared behavior of the decorrelation speed in rapidity.

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References

- [1] F. Gelis, E. Iancu, J. Jalilian-Marian and R. Venugopalan, *Ann. Rev. Nucl. Part. Sci.* **60**, 463 (2010), [arXiv:1002.0333 [hep-ph]]; T. Lappi, *Int. J. Mod. Phys. E* **20**, 1 (2011), [arXiv:1003.1852 [hep-ph]].
- [2] I. Balitsky, *Nucl. Phys.* **B463**, 99 (1996), [arXiv:hep-ph/9509348]; Y. V. Kovchegov, *Phys. Rev.* **D60**, 034008 (1999), [arXiv:hep-ph/9901281].
- [3] A. Dumitru, J. Jalilian-Marian, T. Lappi, B. Schenke and R. Venugopalan, *Phys. Lett.* **B706**, 219 (2011), [arXiv:1108.4764 [hep-ph]].
- [4] PHENIX, A. Adare *et al.*, *Phys. Rev. Lett.* **107**, 172301 (2011), [arXiv:1105.5112 [nucl-ex]]; E. Braidot, arXiv:1102.0931 [nucl-ex].
- [5] K. Tuchin, *Phys. Lett.* **B686**, 29 (2010), [arXiv:0911.4964 [hep-ph]]; J. L. Albacete and C. Marquet, *Phys. Rev. Lett.* **105**, 162301 (2010), [arXiv:1005.4065 [hep-ph]].
- [6] J. Kuokkanen, K. Rummukainen and H. Weigert, *Nucl. Phys.* **A875**, 29 (2012), [arXiv:1108.1867 [hep-ph]].
- [7] F. Dominguez, C. Marquet, B.-W. Xiao and F. Yuan, *Phys. Rev.* **D83**, 105005 (2011), [arXiv:1101.0715 [hep-ph]].
- [8] E. Iancu and D. Triantafyllopoulos, *JHEP* **1111**, 105 (2011), [arXiv:1109.0302 [hep-ph]]; E. Iancu and D. Triantafyllopoulos, *JHEP* **1204**, 025 (2012), [arXiv:1112.1104 [hep-ph]].
- [9] T. Lappi and H. Mäntysaari, arXiv:1207.6920 [hep-ph].
- [10] F. Gelis, T. Lappi and R. Venugopalan, *Phys. Rev.* **D79**, 094017 (2008), [arXiv:0810.4829 [hep-ph]]; T. Lappi, *Acta Phys. Polon.* **B40**, 1997 (2009), [arXiv:0904.1670 [hep-ph]].
- [11] A. Dumitru, F. Gelis, L. McLerran and R. Venugopalan, *Nucl. Phys.* **A810**, 91 (2008), [arXiv:0804.3858 [hep-ph]]; K. Dusling, F. Gelis, T. Lappi and R. Venugopalan, *Nucl. Phys.* **A836**, 159 (2010), [arXiv:0911.2720 [hep-ph]].
- [12] T. Lappi, S. Srednyak and R. Venugopalan, *JHEP* **01**, 066 (2010), [arXiv:0911.2068 [hep-ph]]; A. Dumitru *et al.*, *Phys. Lett.* **B697**, 21 (2011), [arXiv:1009.5295 [hep-ph]]; K. Dusling and R. Venugopalan, arXiv:1201.2658 [hep-ph].